Analysis of Dynamics of Hub Bearings under Moment Loads

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Stiffness of hub bearings is an important factor for vehicle handling stability. Analysis and measurements for the hub bearing stiffness have been done only under non-rotating inner ring conditions but not done under rotating inner ring conditions. Hence, the authors have examined the rotating hub bearing stiffness by using a bearing dynamic analysis system “IBDAS”. This examination shows that rotating hub bearing stiffness coincides with the nonrotating one and hysteresis, that is response delay, exists in displacement curves of inner ring alignment angle against varying moment.

1. Introduction

Even as automated driving technology develops and computers control steering, vehicle handling stability continues to be an important aspect. That is because even with the most advanced computer system if the vehicle response to the steering signal is too slow, then the vehicle will have difficulty driving along the desired path. On the other hand, the stiffness of hub bearings that support the tires of the vehicle is one of the factors that affects vehicle handling stability\(^1\), \(^2\). Therefore, designing the stiffness of hub bearings appropriately is an important subject.

In general, measuring stiffness of hub bearings is done without rotating the shaft\(^2\). Although, it is expected that stiffness with a rotating shaft does not differ from the static rigidity, it is not confirmed, yet. To determine if this is valid, dynamic analysis of the relationship between the shaft rotation speed and stiffness of hub bearings was established using an integrated dynamic analysis system “IBDAS”\(^3\), which NTN has been developing for the analysis of roller bearings\(^4\). In this article, the results obtained from these dynamic analyses are reported.

2. Product to Be Analyzed

A hub bearing consists of two rows of balls and a cage, as shown in Fig. 1. The inner ring on the inboard side is fixed to the hub ring and a certain axial preload is applied to the balls. Therefore, in this report, the rotation system configured with the double row angular contact ball bearing specified in Table 1 is calculated.

<table>
<thead>
<tr>
<th>Table 1: Bearing specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ball diameter (mm)</td>
</tr>
<tr>
<td>Number of balls</td>
</tr>
<tr>
<td>Pitch diameter of ball set (mm)</td>
</tr>
<tr>
<td>Contact angle (deg)</td>
</tr>
<tr>
<td>Distance between rows of balls (mm)</td>
</tr>
<tr>
<td>Preload (N)</td>
</tr>
</tbody>
</table>

* CAE R&D Center
Table 2: Operation condition

<table>
<thead>
<tr>
<th>Vehicle speed (km/h)</th>
<th>10, 50, 100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment (N mm)</td>
<td>−600×10³ - 600×10³</td>
</tr>
<tr>
<td>Axial load $F_a$ (N)</td>
<td>−1,890 - 1,890</td>
</tr>
<tr>
<td>Radial load $F_r$ (N)</td>
<td>1,820 - 3,750</td>
</tr>
<tr>
<td>Load acting point (mm)</td>
<td>10.4</td>
</tr>
<tr>
<td>Load variation frequency (Hz)</td>
<td>1, 10, 50</td>
</tr>
</tbody>
</table>

Fig. 2 Load acting position and coordinate system of analysis

3. Method of Analysis

Temporal waveform of the inclining angle of the operating hub bearing under moment load was calculated using an integrated dynamic analysis system “IBDAS”, which is NTN’s proprietary development for analysis of roller bearings.

The degrees of freedom of motion set for the analytical model and the constraint conditions are as shown in Table 3. The outer ring of the bearing is fixed in the space and the inner ring has degrees of freedom of motion excluding rotation. The inner ring was given the weight and the inertial moment of hub ring. The balls and the cage have 3 translational and 3 rotating degrees of freedom. IBDAS can take into account the elastic deformation of the cage using the mode synthesis method. This analysis gives a total of 56 deformation modes to each cage.

Table 3: Degrees of freedom of motion and constraint conditions within the analytical model

<table>
<thead>
<tr>
<th>Outer ring</th>
<th>Degrees of freedom: None (fixed)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner ring</td>
<td>Degrees of freedom: 3 translational, 2 rotating Constraint: rotation at a constant speed</td>
</tr>
<tr>
<td>Ball</td>
<td>Degrees of freedom: 3 translational, 3 rotating</td>
</tr>
<tr>
<td>Cage</td>
<td>Degrees of freedom: 3 translational, 3 rotating Elastic deformation: 56 modes</td>
</tr>
</tbody>
</table>

In order to dynamically analyze the motion of the ball bearing system with the above degrees of freedom, normal force and tangential force at the contact between the ball/raceway and the ball/cage must be appropriately calculated. The following are the respective methods of calculation. In addition, moments of the balls generated by various forces are also calculated, as required.

3.1 Contact Point Between Balls/Raceway

Normal force is obtained assuming that the contact pressure follows Hertz Theory. In calculating tangential force, the distribution of contact pressure on the major axis of the contact ellipse and sliding velocity are considered as shown in Fig. 3 for an appropriate expression of 3-dimensional ball motion. The specific calculating method is as follows:

The normal load on one of the $n$th pieces sliced along the major axis of the contact ellipse under hertzian pressure can be obtained by integrating the contact pressure along the direction of the minor axis, then integrating it along the major axis within the range of the particular sliced piece and can be expressed in the following equation:

$$F_{Nj} = \frac{3F_N}{2n} \left[ 1 - \frac{12\left( j - 0.5(n-1) \right)^2 + 1}{3n^2} \right]$$  (1)

Where, $F_{Nj}$ is the normal force acted on the $j$th section, $F_N$ is the normal force of the entire contact area and $j$ is the section number (0 to $n-1$).

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For the tangential force, the following 3 types of force elements, which can be assumed in the contact area under oil lubrication, were considered.

1. Traction*1
2. Rolling viscosity resistance*2
3. Oil film force on the rolling direction*3

Since hub bearings are lubricated by grease, this report assumed that the above 3 types of force elements are determined only by the base oil of the grease. The following is an outline of calculation of each force element:

Force acting on each section was calculated using the following equation, assuming that it would act in the opposite direction of the sliding velocity vector of the ball surface against the raceway, \( \mathbf{u}_{\text{slide}} \):

\[
\mathbf{F}_j = -\phi_j \cdot F_j \cdot \frac{\mathbf{u}_{\text{slide}}}{|\mathbf{u}_{\text{slide}}|}
\]

(2)

Where \( F_j \) is the force acting on the ball, \( \phi_j \) is the friction coefficient and the subscript \( j \) represents the \( j^{th} \) section. The arrow on top means that it is a vector. \( \phi_j \) under fluid lubrication was obtained using the calculation method that considers the property of lubricating oil*3, \( \phi_j \) under boundary lubrication was obtained by a function with only slide-roll ratio*4 and \( \phi_j \) under mixed lubrication was obtained by interpolating between those friction coefficients proportionally to the film parameter*5. The friction coefficient greatly depends on the slide-roll ratio even under fluid lubrication, as shown in the calculation example in Fig. 4.

The sliding velocity vector of the ball surface against the raceway, \( \mathbf{u}_{\text{slide}} \), was obtained using the following equation:

\[
\mathbf{u}_{\text{slide}} = \mathbf{v}_b + \mathbf{\omega}_b \times \mathbf{r}_{b\text{ej}} - \left( \mathbf{v}_{\text{race}} + \mathbf{\omega}_{\text{race}} \times \mathbf{r}_{\text{raceej}} \right)
\]

(3)

*1 On two objects in rolling/sliding contact, this is the tangential force which acts on the opposite direction from the rolling direction at the higher speed side and on the same direction as the rolling direction at the lower speed side.

*2 This is the force caused by shear force of oil film which acts in the direction to prevent rolling of two objects in rolling contact, under oil lubrication.

*3 This is the force that acts on the rolling direction due to the oil film pressure that acts on two objects in rolling contact under oil lubrication. This is generated as the center of oil film pressure shifts to the upstream side.

*4 Sliding velocity of the contact surface divided by the average velocity of the contact surface.

Where, \( \mathbf{v}_b \) is the translational velocity vector of the center of the ball, \( \mathbf{\omega}_b \) is the angular velocity vector of the ball, \( \mathbf{r}_{b\text{ej}} \) is the position vector from the center of the ball to the \( j^{th} \) section surface, \( \mathbf{v}_{\text{race}} \) is the translational velocity vector of the center of raceway, \( \mathbf{\omega}_{\text{race}} \) is the angular velocity vector of the raceway, and \( \mathbf{r}_{\text{raceej}} \) is the position vector from the center of the raceway to the \( j^{th} \) section surface. The method for obtaining the position of the section surface within the contact ellipse is based on the Jones method*6.

The sliding velocity vector of the section on the ball surface, \( \mathbf{u}_{\text{slide}} \), has a component of the axial direction in addition to the rolling direction, as shown in Fig. 5. Therefore, there is a force acting in the direction of major axis of the contact ellipse.

![Fig. 5 Example of sliding velocity vector on the ball surface](image)

The rolling viscosity resistance acts in the opposite direction of rolling direction against the ball and the raceway. Since it is the force from the lubricating film, it was assumed that this only occurs under fluid lubrication where lubricating film is formed. And the magnitude of the rolling viscosity resistance \( F_j \) was obtained, using the equations for*10, 11) the case of high pressure viscosity elastic body range (PE) and high pressure viscosity rigid body range (PR). The case of isobaric viscosity rigid body range (IR) is also used selectively, depending on the result of the range determination*10:

\[
F_j = \begin{cases} 
C_j 2.9 R_j \left( \frac{G U_j}{W_j} \right)^{0.648} \frac{W_j^{0.246}}{w} & \text{for PE, PR} \\
0.09 R_j \left( \frac{W_j}{U_j} \right)^{0.509} \frac{W_j}{w} & \text{for IR}
\end{cases}
\]

(4)

Where, \( C_j \) is the coefficient of thermal correction, \( R_j \) is the effective radius, \( G \) is the material parameter, \( U_j \) is the velocity parameter, \( W_j \) is the load parameter, \( w \) is the section width, \( E^* \) is the equivalent elastic modulus and the subscript \( \text{max} \) refers to the section where the contact pressure is the greatest.

The oil film force on the rolling direction \( F_{ip} \) was obtained using the following equation*10 from the rolling viscosity resistance*10:

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\[ F_{py} = \frac{2R_y^3F_{Ri}}{R_{yl}} \]  

(5)

Where, \( R_{yl} \) is the radius from the rotational axis of the ball to the \( j^{th} \) section of the contact area.

As the distribution of forces in the contact ellipse and their 3D effect are put into consideration, the calculation result of rotation and revolution velocity of the balls from IBDAS closely matches\(^9\) the calculation result from Jones Theory\(^9\).

### 3.2 Contact Point Between Balls/Cage

It was assumed that the normal force, \( F_{NC} \), that corresponds to the interference amount, \( \delta_c \), is generated following the Hertz Theory meaning the surface of the cage is divided into finite elements and those nodes geometrically interfere with the balls.

\[ F_{NC} = k_{Hertz} \delta_c^{1.5} \]  

(6)

Where, \( k_{Hertz} \) is the non-linear spring constant in the Hertz Theory.

In calculating tangential force, only force from sliding friction was considered, as sliding is the only force between the balls and the cage. Sliding friction force was obtained by the following equation.

\[ \vec{F}_{IC} = -\mu_s \vec{F}_{NC} \frac{\vec{u}_C}{|\vec{u}_C|} \]  

(7)

Where, \( \mu_s \) is the friction coefficient and \( \vec{u}_C \) is the sliding velocity vector of the ball surface against the cage pocket. For \( \mu_s \), 0.06 was used in this calculation.

### 4. Calculation Result

Fig. 6 shows the calculation result of the inclining angle of the inner ring on the applied moment in each vehicle speed. The load variation frequency is 50 Hz. It revealed that the waveform of the inclining angle of the inner ring on the applied moment has hysteresis. It also revealed that the slopes of the lines for each vehicle speed (compliance) are, although with hysteresis, the same. From these results, it was revealed that the rigidity of hub bearings, which is the inverse number of compliance, is also mutually the same regardless of vehicle speed.

Fig. 7 shows the calculation result of the inclining angle of the inner ring on the applied moment, when the load variation frequency is changed to 3 different levels with vehicle speed of 10 km/h. The slopes of the lines of the inclining angles of the inner ring on the applied moment remain the same, revealing that the rigidity of hub bearings does not depend on the load variation frequency.

On the other hand, hysteresis is also observed in the lines of Fig. 7. The hysteresis of the inclining angle of the inner ring on the applied moment means that the increase of lateral displacement of the body lags a little behind the tire lateral force input from the ground. If lateral displacement of body against steering is considered, it means the peak of the lateral displacement appears a little late. This reduces vehicle handling stability.

Therefore, the effect of driving conditions on the magnitude of hysteresis is determined by examining the difference of the inclining angle at the moment 0.3 kNm. It is defined as the fluctuation range \( W_{IF} \) as shown in Fig. 8.
Fig. 8 Definition of fluctuation range $W_{H}$, which indicates the magnitude of hysteresis of inclining angle of the inner ring against the moment (Points $P_{r}$ and $P_{s}$ are used in Chapter 5)

Fig. 9 shows the fluctuation range of the inclining angle of the inner ring when vehicle speed and load variation frequency are both changed. As shown in Fig. 9, the fluctuation range increased as the vehicle speed reduced and the load variation frequency increased.

Fig. 9 Calculation result of fluctuation range of inclining angle of the inner ring against the moment $W_{H}$, when vehicle speed and load variation frequency are changed (bubble diameter is proportional to fluctuation range, numbers next to bubbles are fluctuation range $W_{H}$ (deg/10^3))

5. Examination

The following discusses the mechanism of how fluctuation range, $W_{H}$, affects the waveform of the inclining angle of the inner ring on the applied moment.

The reason for the fluctuation range is shown in Fig. 10. The force in the direction of the major axis of the contact ellipse created between the ball and the raceway hinders the movement of the ball in the radial plane. This mechanism is described below, focusing on the forces that act on the upper right ball of Fig. 10.

Fig. 10 Overall concept of the normal force and traction that act on the balls from the raceway when the inner ring inclines counterclockwise

The typical state of forces that act on the upper right ball of Fig. 10 is shown in Fig. 11. First, Fig. 11 a) shows the state where the inner ring continues to rotate while the inclining angle of the inner ring remains constant. In this case, since the inner ring is rotating with the same inclining angle of the inner ring, the center of the ball is positioned on the line that connects the centers of the grooves of the inner and outer rings. The normal forces, $F_{N}$ and $F_{N'}$, which act on the ball from the inner/outer rings are aligned on the same axis and remain balanced (here, the centrifugal force is ignored for ease of explanation). The forces that act on the ball from the inner/outer raceways act only in the direction of the roller, not having the surface component in the figure.

Fig. 11 b) shows the state immediately after the inclining angle of the inner ring increases its magnitude in the counter clockwise direction, where the ball transition is not sufficient. In this case, the normal force, $F_{N}$ from the inner ring changes its direction as shown in the figure (here, force, $F_{N'}$ that acts on the ball from the inner ring is ignored for ease of explanation). The resulting force that acts on the ball from the raceway will be toward upper left as shown in the figure.

Next, a case where the inclining angle of the inner ring is the same as Fig. 11 c), but the moment is decreasing and
the inclining angle of the inner ring changes in the clockwise direction is considered. Let us consider the case of P_F as the P_F point in Fig. 8. The force that acts on the point P_F can be described in Fig. 12. Since the forces that act on the ball from the inner/outer rings act in the lower left direction, the ball remains toward to lower left from the line that connects the centers of the inner/outer raceway grooves. The angle of F_{Ni} also becomes smaller. As a result, the magnitude of the moment the ball induces on the inner ring is smaller than Fig. 11 c) meaning the moment at the point P_F in Fig. 8 is smaller than the point P_F.

As seen above, due to the forces acting in the major axis direction of the contact ellipse, the center position of the ball where the inclining angle of the inner ring cannot reach the ball position where the inclining angle of the inner ring is constant. And this force produces the fluctuation range of the inclining angle of the inner ring.

![Diagram showing forces and inclining angles](image)

**Fig. 12** Normal force and traction that act on the balls from the raceway when the inner ring inclines clockwise

The fluctuation range, W_H, correlates with the magnitude of the force in the major axis direction of the contact ellipse that acts on the ball, as mentioned above. This magnitude of force greatly depends on the slide-roll ratio as shown in Fig. 4. The slide-roll ratio on the ball surface, S_j, is a ratio of the sliding velocity on the ball surface, \|v_{slide}\|, over the rolling velocity of the contact area, \|v_{roll}\|, and is given in the following equation:

\[
S_j = \frac{\|v_{slide}\|}{\|v_{roll}\|}
\]  

(8)

For simplification, slide-roll ratio at the pure rolling position (no sliding in the rolling direction) in the contact area between the ball and raceway is considered. In this case, the sliding velocity and the rolling velocity are proportional to the load variation frequency, f_{load}, and the vehicle speed, u_v, respectively. The slide-roll ratio, S_j, can be expressed as follows:

\[
S_j \propto \frac{f_{load}}{u_v}
\]  

(9)

Therefore, it can be concluded that the increase of f_{load} and decrease of u_v result in the increase of the slide-roll ratio S_j, friction coefficient \( \phi_j \), force and fluctuation range W_H, which corresponds with the trend in Fig. 9.

The above is a discussion of pure rolling position within the rolling contact area. At the position where sliding velocity exists in the rolling direction, the numerator of Equation (8) includes the sliding velocity component in the rolling direction which slightly reduces the impact of f_{load} on S_j. However, since the sliding velocity component in the major axis direction increases, the forces in the major axis still increases.
6. Conclusion

The rigidity of hub bearings with inner ring rotation was analyzed using the dynamic analysis system, IBDAS. As a result, it was verified that the rigidity of hub bearings does not depend on the rotational speed of the inner ring and variation frequency of the moment. It was also verified that the waveform of the inclining angle of the inner ring over the varying moment has hysteresis. The reason for this hysteresis is because of the traction in the direction of major axis of the contact ellipse created between the ball and the raceway. Increasing the load variation frequency and decreasing the vehicle velocity (decreasing the rotational speed of the shaft) increases traction in the major axis direction of the contact ellipse, ultimately increasing the hysteresis range.

The above hysteresis can be interpreted as the delay of response of hub bearing angle to the change of moment. In order to increase vehicle handling stability, it is desirable to reduce this delay. We will continue further analysis using CAE technology to developing hub bearings with reduced hysteresis.

References

5) NTN Corp. Hub Bearings Catalog, CAT. No. 4601/J.