Evaluation of Rolling Contact Fatigue by X-ray Diffraction Ring

1. Introduction

It has been well documented (within many reports) that fatigue associated with microstructural changes, hardness changes, and the formation of residual stress can be observed under the raceway of rolling bearings,\(^1\), \(^2\).

This is an attempt to estimate the residual life of rolling bearings, to determine the risk of damage to rolling bearings. Previously, three parameters (residual stress, half width and retained austenite assuming plane stress condition) obtained by X-ray stress measurement device were used in the research. However, since the fatigue of rolling bearings varies depending on the operation conditions, it is difficult to accurately estimate the remaining life.

Recently, an analytical device (which can detect Debye rings) was developed and is currently being studied for use in material evaluation\(^3\). Specifically, Debye rings can be used for measuring stress with a single incident angle (cos \(\alpha\) method\(^4\)). With the cos \(\alpha\) method, the stress can be obtained quickly compared with the conventional methods. In addition, if Sasaki-Hirose method\(^5\), which is an extension of cos \(\alpha\) method, tri-axial stress measurement is also possible. Furthermore, information on the orientation of crystal grains can be obtained from the intensity distribution of the Debye rings (obtained on the detector). By using this information obtained by unconventional methods, it should be possible to accurately estimate residual life from the complex mechanism of rolling contact fatigue.

In this paper, we will study the relation between the crystal orientation of martensite (obtained from the Debye rings) and tri-axial residual stress (obtained by Sasaki-Hirose method) in the progression of rolling-contact fatigue of two cylindrical specimens.

2. Theory of Tri-Axial Stress Measurement by X-ray

2.1 Stress Measurement Method with Debye rings

When an X-ray is incident to an object of crystalline structure, a diffraction phenomenon occurs within the crystal that satisfies Bragg conditions, as indicated by the following equation (1).

\[
2d \cdot \sin \theta = n \lambda \tag{1}
\]
Where,
\( d \): Lattice spacing
\( \theta \): Bragg angle
\( n \): Integer
\( \lambda \): Wavelength of X-ray

In general, metallic materials (for industrial use) are often poly-crystalline possessing a random crystalline orientation. When tensile stress is applied to these types of materials, the lattice spacing of the crystal grains vary (as shown in Fig. 1) as well as the diffraction angle of each crystal grain changes. Therefore, X-ray stress measurement is accomplished by calculating stress using the difference between the diffraction angles (due to the orientation of crystals).

If the specimen has a crystal grain suitable for X-ray stress measurement (the diameter of crystal grain is 30 \( \mu \)m or less in the state without orientation or aggregate structure), diffraction X-ray is generated within a conical shape centered around incident X-ray (shown in Fig. 2). The diffraction X-ray is measured by a two-dimensional “Debye Rings” detector (as cross-sections of a cone).

For the X-ray stress measurement theory using whole Debye rings, the \( \cos \alpha \) method (proposed by Taira, etc.) and a method using an imaging plate (IP) as a detector (proposed by Yoshioka, etc.), and Sasaki, etc.). The \( \cos \alpha \) method is an analysis method that assumes a plane stress condition; the Sasaki-Hirose method is a tri-axial stress measurement method.

### 2.2 Sasaki-Hirose Method

The strain \( \varepsilon_a \) to the central angle \( \alpha \) of Debye ring can be expressed as the equation (2) using the radius of Debye ring \( R_a \).

\[
\varepsilon_a = \frac{1}{2} \left[ \frac{2 \theta_0 - \pi}{\tan^{-1} \left( \frac{R_a}{CL} \right)} \right] \cot \theta_0 \quad \text{(2)}
\]

Where,
\( \theta_0 \): Bragg angle without strain
\( R_a \): Radius of Debye ring at the position \( a \)
\( CL \): Distance of X-ray from the radiation point to the detector

The relation between \( \varepsilon_a \) and stress can be expressed by the following equation in the coordinate system in Fig. 3.

\[
\varepsilon_a = \sigma_x \frac{1}{E} \left[ n_1^2 - \nu (n_2^2 + n_3^2) \right] + \sigma_y \frac{1}{E} \left[ n_2^2 - \nu (n_1^2 + n_3^2) \right] + \sigma_z \frac{1}{E} \left[ n_3^2 - \nu (n_1^2 + n_2^2) \right] + \tau_{xy} \frac{2(1+\nu)}{E} n_1 n_2 + \tau_{xz} \frac{2(1+\nu)}{E} n_1 n_3 + \tau_{yz} \frac{2(1+\nu)}{E} n_2 n_3 \quad \text{(3)}
\]

Where,
\( E \): Young’s modulus of X-ray
\( \nu \): Poisson’s ratio

In addition, \( n_1 \) to \( n_3 \) are direction cosine of \( \varepsilon_a \) and can be expressed by the following equations:

\[
n_1 = \cos \eta \sin \psi_0 \cos \phi_0 - \sin \eta \cos \psi_0 \cos \phi_0 \cos \alpha - \sin \eta \sin \phi_0 \sin \alpha \quad \text{(4)}
\]

\[
n_2 = \cos \eta \sin \psi_0 \sin \phi_0 - \sin \eta \cos \psi_0 \sin \phi_0 \cos \alpha + \sin \eta \cos \phi_0 \sin \alpha \quad \text{(5)}
\]

\[
n_3 = \cos \eta \cos \psi_0 + \sin \eta \sin \psi_0 \cos \alpha \quad \text{(6)}
\]
Evaluation of Rolling Contact Fatigue by X-ray Diffraction Ring

Where,
\( \eta \) : Supplementary angle of Bragg angle \\
\( \psi_0 \) : X-ray incident angle \\
\( \phi_0 \) : Angle made by incident X-ray and X-axis

Then, to obtain each stress component from the equation (3), strains at the central angles \( \pi + \alpha \), \( \pi - \alpha \) and \( -\alpha \) are expressed as \( \varepsilon_{\pi + \alpha} \), \( \varepsilon_{\pi - \alpha} \) and \( \varepsilon_{-\alpha} \), respectively, with the following parameters newly defined.

\[
a_{1(\phi_0)} = \frac{1}{2} [(\varepsilon_{\pi + \alpha} - \varepsilon_{\pi - \alpha}) + (\varepsilon_{\pi - \alpha} - \varepsilon_{\pi + \alpha})] \quad \text{(7)}
\]

\[
a_{2(\phi_0)} = \frac{1}{2} [(\varepsilon_{\pi + \alpha} - \varepsilon_{\pi - \alpha}) - (\varepsilon_{\pi - \alpha} - \varepsilon_{\pi + \alpha})] \quad \text{(8)}
\]

From equations (3) to (8), \( a_1 \) and \( a_2 \) at \( \phi_0 = 0 \) are expressed as follows:

\[
a_{1(0)} = \frac{-1 + \nu}{E} \left[ (\sigma_x - \sigma_z) \sin 2\psi_0 + 2 \tau_xz \cos 2\psi_0 \right] \times \sin 2\eta \cos \alpha \quad \text{(9)}
\]

\[
a_{2(0)} = \frac{2 (1 + \nu)}{E} \left[ \tau_x \sin \psi_0 + \tau_{yz} \cos \psi_0 \right] \times \sin 2\eta \sin \alpha \quad \text{(10)}
\]

When \( \psi_0 = 0 \) in equations (9) and (10), namely, when X-ray is incident perpendicularly onto the specimen (perpendicular incident), the following equations can be obtained with the shear stresses \( \tau_{xz} \) and \( \tau_{yz} \):

\[
\tau_{xz} = -\frac{E}{2 (1 + \nu)} \frac{1}{\sin 2\eta} \left( \frac{\partial a_{1(0)}}{\partial \cos \alpha} \right) \quad \text{(11)}
\]

\[
\tau_{yz} = \frac{E}{2 (1 + \nu)} \frac{1}{\sin 2\eta} \left( \frac{\partial a_{2(0)}}{\partial \sin \alpha} \right) \quad \text{(12)}
\]

In addition, equations (9) and (10) indicate that \( a_1 \) and \( a_2 \) are linear to \( \cos \alpha \) and \( \sin \alpha \), respectively, with the slope to be expressed in the following equations:

\[
\left( \frac{\partial a_{1(0)}}{\partial \cos \alpha} \right) = -\frac{1 + \nu}{E} \left[ (\sigma_x - \sigma_z) \sin 2\psi_0 + 2 \tau_xz \cos 2\psi_0 \right] \times \sin 2\eta \quad \text{(13)}
\]

\[
\left( \frac{\partial a_{2(0)}}{\partial \sin \alpha} \right) = \frac{2 (1 + \nu)}{E} \left[ \tau_x \sin \psi_0 + \tau_{yz} \cos \psi_0 \right] \times \sin 2\eta \quad \text{(14)}
\]

In equations (13) and (14), since \( \tau_{xz} \) and \( \tau_{yz} \) are already obtained by equations (11) and (12), when \( \psi_0 \neq 0 \), \( \sigma_x - \sigma_z \) and \( \tau_{yz} \) can be obtained by the following equations:

\[
\sigma_x - \sigma_z = -\frac{E}{1 + \nu} \frac{1}{\sin 2\eta} \frac{1}{\sin 2\psi_0} \left( \frac{\partial a_{1(0)}}{\partial \cos \alpha} \right) - 2 \tau_xz \cot 2\psi_0 \quad \text{(15)}
\]

\[
\tau_{yz} = \frac{E}{2 (1 + \nu)} \frac{1}{\sin 2\eta} \frac{1}{\cos \psi_0} \left( \frac{\partial a_{2(0)}}{\partial \sin \alpha} \right) - \tau_{xz} \cot \psi_0 \quad \text{(16)}
\]

\( \sigma_x - \sigma_z \) can be calculated when the relation of equation (15) at \( \phi_0 = \pi/2 \) rad (90°) is used.

\( \sigma_x \) is obtained from the following equation.

\[
\sigma_x = \frac{E}{1 - 2\nu} (\varepsilon_{\pi} - \varepsilon_{x}) \quad \text{…………………………(17)}
\]

\( X \) can be expressed by the following equation.

\[
X = \frac{2 (1 + \nu)}{E} \left( \tau_{xy} n_1 n_2 + \tau_{yz} n_2 n_3 + \tau_{xz} n_3 n_1 \right) + \frac{1}{E} (\sigma_x - \sigma_z) [n_1^2 - \nu (n_2^2 + n_3^2)] + \frac{1}{E} (\sigma_y - \sigma_z) [n_2^2 - \nu (n_2^2 + n_1^2)] \quad \text{…………………………(18)}
\]

Since each stress component and direction cosine are already known in equation (18), the value of \( \sigma_z \) can be obtained. In addition, equation (17) includes a term of \( \varepsilon_{\pi} \), and since \( \varepsilon_{\pi} \) can be obtained for each central angle of Debye ring, \( \sigma_z \) is determined as their average.

The above is the tri-axial stress analysis method by the Sasaki-Hirose method and 6 stress components can be obtained by three X-ray irradiations indicated by Fig. 5.

3. Experiment Conditions

3.1 Two-Cylinder Test Conditions

Fig. 4 shows the schematic overview of the two-cylinder testing machine and Table 1 shows the test conditions. The test was conducted under the pure rolling conditions. No. 1 to No. 3 are conducted under the conditions of using a specimen of small surface roughness for the driving side (mirror surface drive), No. 4 is conducted under the condition of using a specimen of large surface roughness for the driving side (rough surface drive). In the two-cylinder test, it is known that the damage progression of the cylinder in the mirror side is faster with rough surface drive rather than mirror surface drive. No. 10, 11, and we have investigated its effect on the evaluation of fatigue by X-ray.

The two-cylinder test specimens are made of JIS SUJ2 with standard heat treatment. The shape is cylindrical of \( \varphi 40 \times 12 \)mm and the radii of curvature in the axial direction are indicated in Table 1. No. 1 is a boundary lubricating condition with the maximum contact stress \( (P_{\text{max}}) \) of 2.77 GPa and the oil film parameter \( (\Lambda) \) of 0.3. No. 2 has mirror surface finish for both specimens (driving/rolling sides) to comply with the elastohydrodynamic lubrication condition \( (\Lambda > 3) \). No. 3 uses mirror surface finish for the driving side specimen and ground surface for the following side specimen, similar to No. 1, however, \( P_{\text{max}} \) is 2.20 GPa, which is a lower surface pressure than No. 1.

Lubrication oil was supplied by contacting a felt pad
containing additive-free turbine oil (ISO VG32) to the specimens. The rotational speed of the driving side specimen was set at 500 min⁻¹.

3.2 X-ray Measurement Conditions

We used μ-X360 (made by Pulstec Industrial Co., Ltd.) as the X-ray diffraction ring analyzer. This equipment can analyze the Debye rings (as the diffraction X-ray is detected in two dimensions).

Table 2 shows the X-ray measurement conditions. We used the coordinate system (shown in Fig. 5) for the measurement, and the Sasaki-Hirose method described in Chapter 2 for the stress analysis. The irradiation range of the X-ray is φ2, which is smaller than the major axis of the contact ellipse (2.96 mm) calculated from the test conditions, therefore, only the rolling contact area can be evaluated.

The depth that the X-ray can penetrate in the material (X-ray penetration depth) is given by the following equation 12).

$$T_a = \frac{1}{\mu} \frac{\cos2\beta \cos^2 \psi_0 \cos \psi_0 \sin2\beta \sin \psi_0 \cos \alpha}{(1 + \cos2\beta \cos \psi_0 \sin2\beta \sin \psi_0 \cos \alpha)} \quad (19)$$

Where,

$T_a$ : Penetration depth of diffraction ray at the position $\alpha$

$\mu$ : Ray absorption coefficient of iron against Cr-Kα ray 13) (889.76 cm⁻¹)

By using equation (19), the relationship between the incident angle $\psi_0$ and the average penetration depth is depicted in Fig. 6. In this paper, as we set the incident angle of X-ray as 0 rad (0°) and 0.524 rad (30°), the average penetration depths of X-ray are 5.4 μm and 4.6 μm, respectively. Therefore, the X-ray measurement results that we describe later indicate information on the organization and stress variation up to approx. 5 μm from the surface of the specimens.
3.3 Evaluation Indices

As the evaluation indices for rolling contact fatigue, parameter \( S/S_0 \), which is defined for the purpose of quantifying change of Debye rings caused by the equivalent stress (Mises stress) \( \sigma_{eq} \) and the crystal orientation, is used. \( S \) is the standard deviation of diffraction intensity to the central angle of Debye ring and \( S_0 \) is the value of \( S \) before the test. With the Debye ring with heterogeneous intensity distribution as shown in Fig. 7 (a), the peak intensity against \( \alpha \) is shown in Fig 7 (b) and its standard deviation is indicated by \( S \) in the right diagram. On the other hand, \( \sigma_{eq} \) is expressed by equation (20).

\[
\sigma_{eq} = \sqrt{\frac{1}{2} \left( (\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 \right) + \left( \sigma_x^2 + \sigma_y^2 + \sigma_z^2 \right)}
\]

Fig. 7 Definition of parameter \( S \)

4. Results and Observations

4.1 Effect of the Oil Film Parameter

Fig. 8 shows the relation between \( \sigma_{eq} \) and the number of cycles for driving specimens of No. 1 and No. 2. \( \sigma_{eq} \) of No. 2, which is the elastohydrodynamic lubrication condition, exhibits almost no change even when the number of cycles becomes large; however, \( \sigma_{eq} \) of No. 1 exhibits the maximum value at the beginning of the rolling contact, increasing to 1134 MPa, which is close to the yield stress of SUJ2. Then, \( \sigma_{eq} \) gradually decreased along with peeling. Fig. 9 shows the condition of the raceway at the end of the test. This peeling effect was already observed from the early stage (driving cycle: 12 x 10^4).

Fig. 10 shows the Debye rings obtained from the specimens before and after the test. With No. 1, heterogeneous Debye rings were observed after the test. Fig. 11 shows the relation between the driving cycles and \( S/S_0 \) of No. 1 and No. 2. \( S/S_0 \) of No. 2 did not change until the end of the test, however, \( S/S_0 \) of No. 1 increased as the driving cycle increased. This indicates that the rolling contact fatigue (under the boundary lubrication condition) is a phenomenon with orientation of martensite crystal grains.

Fig. 8 Relationship between von Mises stress \( \sigma_{eq} \) and number of cycles for driving specimens of No.1 and No.2 tests

Fig. 9 Peeling for driving specimens of No.1 test after RCF of 48x10^4 cycles
4.2 Impact of Load

Fig. 12 shows the change of $\sigma_{eq}$ and $S/S_0$ against driving cycles of No. 1 and No. 3. The behavior of $\sigma_{eq}$ and $S/S_0$ was almost the same between No. 1 and 3, with peeling observed almost at the same time.

The reason why there was no difference in formation of $\sigma_{eq}$, behavior of crystal orientation and timing of peeling (even with different loads) is because there was no difference in severity of asperity contact between the two. $R_{dq}$, which indicates severity of asperity contact, exceeds 0.175 rad (10°) before the test for following side cylinder in No. 1 and 3. This value is much higher than $R_{dq}$ of the regular bearings (0.017 ~ 0.070 rad). Contact pressure generated at the asperity contact area of on the surface of large $R_{dq}$ is estimated to exceed the limit that the SUJ2 materials can support in any load conditions. Therefore, increase of load does not affect the local contact pressure but only to the increase of real contact area. This is also verified by the X-ray analysis. This is the reason why the load did not affect the formation of $\sigma_{eq}$ and behavior of crystal orientation.

4.3 Change of X-ray Measured Values within the Low-Driving Cycle Range

The fact that $\sigma_{eq}$ and the shape of the Debye rings in the boundary lubrication condition quickly change in the driving cycle range up to $10^5$ has been shown above. In the following, we describe the analysis results of driving cycle range up to $10^5$.

Fig. 13 shows change of $\sigma_{eq}$ and $S/S_0$ when rolling contact test was conducted up to $5 \times 10^4$ under the same conditions as No. 1. $\sigma_{eq}$ increased from 533 MPa of before the test to 1084 MPa at $10^5$ driving cycles and stayed mostly constant in the range of 1080 to 1200 MPa up to $5 \times 10^4$. On the other hand, $S/S_0$ showed little change up to $10^5$ and then increased after $10^5$ revealing the fact that the rolling contact fatigue progresses with generation of residual stress then crystal orientation.
4.4 Impact of Rough Surface Drive

Fig. 14 shows the change of $\sigma_{eq}$ and $S/S_0$ for mirror surface drive cycle of No. 1 and rough surface drive cycle of No. 4. $\sigma_{eq}$ and $S/S_0$ exhibited similar behavior in No. 1 and 4 up to the cycles $5 \times 10^4$, then No. 4 showed larger increase of $S/S_0$ thereafter. In addition, peeling was observed from around $5 \times 10^4$ both for No. 1 and 4; however, the progress was faster for No 4 showing significant difference at the cycles of $5 \times 10^5$ in the number and magnitude of damages on the raceway as shown in Fig. 15. That is, rough surface drive promotes significant crystal orientation after peeling starts and the progress of peeling was also faster.

These results show that the progressions of rolling contact fatigue for No. 1 and 4 are different after cracking occurs. Kaneta et al.\textsuperscript{[11]} indicate that the behavior of crack opening can vary between the driving side and following side when cracks and lubricating oil are present on the raceway and this is considered to be the reason why the progressions of peeling in No. 1 and 4 are different. We will study the mechanism of generation and growth of cracks, as well as the mechanism of the formation of crystal orientation, in more detail.
5. Conclusion

We have evaluated the progression of the rolling contact fatigue (under boundary lubrication and fluid lubrication conditions) by the equivalent stress (Mises stress) $\sigma_{eq}$ obtained by X-ray diffraction ring analyzer and $S/S_0$, which indicates the heterogeneity of the Debye rings. The following is the summary of the results:

1) $\sigma_{eq}$ of the rolling contact surface increases closer to the elastic limit of SUJ2 in early stage of around $10^3$.
2) After a rapid increase up to the cycles around $10^3$, $\sigma_{eq}$ remained almost constant until small peeling started.
3) In this experiment, the crystal orientation started after the end of the significant increase of $\sigma_{eq}$, around the point of $10^3$.
4) As the peeling grows, $\sigma_{eq}$ decreases; however, the crystal orientation continues to progress.
5) $\sigma_{eq}$ and crystal orientation behavior did not change, even if the specimens of driving and following sides are switched up to the point peeling started; however, crystal orientation was more significant with the rough driving after peeling started.

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Reference

1) For example, Hiroshi Muro, Noriyuki Tsushima and Masatoshi Tokuda, Changes of Residual Stress due to Rolling Contact, Journal of the Society of Materials Science, Japan, 18, 190, (1969) 615-619.

Photo of authors