Improved Method of Rolling Bearing Fatigue Life Prediction Under Edge Loading Conditions

1. Foreword

More than half a century has passed since Lundberg and Palmgren (referred to as LP in the following) established a theory on the life of rolling bearings \(^{1,2}\), and problems with the theory such as its lack of capability of considering shear force (tangential force) on the surface caused by slip or the initial or residual stress due to fit have been pointed out, with correction to these problems having been reported in many ways. \(^{3-6}\) However, these problems occur only when the theory is used under special conditions, and in most cases, the subjects above do not pose problems. For this reason, designing and examination based on the LP theory is still in practice now from the viewpoint of respecting past track records also.

On the other hand, the design of crowning is important in designing roller bearings. When an imbalance in sharing of load exists between the edge section and the middle section or when misalignment exists, that region will be damaged in a short time should a greater load be exerted on either of both. To avoid a problem of this kind, it is necessary to provide a suitable crowning.

The method of Ito and Sugiura \(^{7}\) is well-known as a method of determining a crowning profile. On the basis of results of various life tests conducted with the amount of crowning of full-crowning rollers being varied, they used the method of Harris \(^{8}\) and Moyer et al. \(^{9}\), proposing an optimum crowning design method. However, this method cannot be applied to (a) the edge load problem occurring in the necking section for grinding on a rolling surface and to (b) the one occurring in the joint between the cylinder section and the crowning section in cut crowning as shown in Fig. 1. Other methods \(^{10,11}\) of determining a crowning profile have been suggested; however, those methods are intended to determine crowning profiles to reduce contact pressure and stress, not intended to calculate the bearing life.

Recently, as is seen in the theory of Ionnides and Harris (referred to as IH in the following) \(^{12}\), various methods have been suggested that determine the life of the entire bearing by dividing the surface loaded by stress into finite pieces, calculating the life of each, and then combining them. These methods are considered to be applicable to any stress distribution condition and should be applicable to the life projection for cases where edge load occurs; however, this has not been ascertained adequately.
In actual designing, machining restrictions and cost problems often prevent one from defining an optimum crowning shape, forcing one to determine a crowning shape within the performance limit of the current processing machine. It is necessary to determine whether the life of the bearing designed in this way can meet the requirement; in doing so, if one can obtain a tool to predict the bearing life accurately, such tools are extremely useful. This paper discusses a method of predicting the life under the effects edge loading that can handle such problem.

2. Rolling fatigue test conducted by Ito and Sugiura

Table 1 shows the result of the rolling fatigue test conducted by Ito and Sugiura. They conducted the rolling fatigue test in which they changed the crowning radius of a full-crowning cylindrical roller with $\phi 12 \times 12$ and the load.

Fig. 2 shows an example of the contour of the amplitude of subsurface shearing stress that occurred this test. The figure shows the half of the contour in the axial direction for the reason of the symmetry. Fig. 3 shows the coordinate system used in the calculation of the shearing stress amplitude. The horizontal axis in Fig. 2 denotes the direction of the axis of the roller and the vertical axis the direction of the depth, with the contour lines of stress amplitude plotted in the figure. Photo (a) shows a state of contact without edge load, exhibiting a stress concentration in the middle of the axial direction (at $x = 6$). Photo (b) shows a concentration of stress both at the edge ($x = 0.5$) and in the middle. Photo (c) shows the result of calculation of stress on a roller without a crowning, exhibiting a remarkable concentration of stress at the edge. Photo (b) shows a weaker concentration of stress in the middle than that in Photo (a), exhibiting a relatively uniform stress distribution.

3. Prediction based on the current life model

The current life prediction model is applied to the results of the experiment described above to ascertain the prediction accuracy of the model. This paper discusses the four life models shown below. In the following analysis, indexes $e$, $c$, and $h$ are coefficients, being 9/8, 31/3, and 7/3 for a roller bearing, respectively.

Model-A

This is a model based on the concept of the LP theory, but it does not use the method of stress calculation in the LP theory; it calculates the life by

<table>
<thead>
<tr>
<th>Test No.</th>
<th>V1</th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
<th>V6</th>
<th>V7</th>
<th>V8</th>
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<tbody>
<tr>
<td>Load (kN)</td>
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<td>17.35</td>
<td>11.76</td>
<td>9.8</td>
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<td></td>
<td></td>
<td></td>
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<tr>
<td>Crowning radius (mm)</td>
<td>300</td>
<td>480</td>
<td>890</td>
<td>1200</td>
<td>$\infty$</td>
<td>480</td>
<td>890</td>
<td>1200</td>
</tr>
<tr>
<td>$L_0 \times 10^4$</td>
<td>560</td>
<td>1240</td>
<td>930</td>
<td>900</td>
<td>400</td>
<td>410</td>
<td>2200</td>
<td>4010</td>
</tr>
<tr>
<td>$L_0 \times 10^4$</td>
<td>2160</td>
<td>4640</td>
<td>2360</td>
<td>1640</td>
<td>745</td>
<td>1330</td>
<td>5100</td>
<td>7250</td>
</tr>
<tr>
<td>Weibull slope</td>
<td>1.4</td>
<td>1.4</td>
<td>2.0</td>
<td>3.2</td>
<td>3.0</td>
<td>1.6</td>
<td>2.2</td>
<td>3.2</td>
</tr>
<tr>
<td>Edge failure/test frequency</td>
<td>0/14</td>
<td>0/15</td>
<td>7/15</td>
<td>12/15</td>
<td>13/15</td>
<td>3/15</td>
<td>6/15</td>
<td>8/15</td>
</tr>
</tbody>
</table>
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means of equation (1) on the basis of the maximum value among the entire shearing stress distribution determined by numerical calculation and the depth of that maximum value. \( S \) denotes the survival probability, \( \tau_0 \) the maximum amplitude of shearing stress, \( z_0 \) the depth of \( \tau_0 \), \( V \) the stress volume, \( a \) the contact width, \( \ell \) the perimeter length of the rolling surface, and \( N \) the number of times of loading.

\[
\log \frac{1}{S} = \frac{\tau_0}{z_0} \cdot N^\epsilon \cdot V = \frac{\tau_0}{z_0} \cdot N^\epsilon \cdot a \cdot \ell \cdots \cdots \cdots \ (1)
\]

**Model-B, and -C**

These are methods based on the IH theory, in which the subsurface portion is divided (see Fig. 4), and the life in the domain \( V_r \) in which the roller undergoes stress greater than the fatigue limit is calculated to determine the life as a whole. The calculation equation is given by equation (2). \( \tau_u \) denotes the stress value of each slice, \( \tau_u \) the fatigue limit, and \( V \) the volume of each slice. With model B, it is assumed that the fatigue limit \( \tau_u \) is 350 MPa, and with model C, it is assumed that the fatigue limit does not exist. The stress volume \( V_r \) of model C is assumed to be the same as the stress volume at a fatigue limit of 350 MPa. Although the IH theory does not explicitly show the stress that provides the reference for life calculation, the shearing stress amplitude is used in this paper as in the LP theory. \( z' \) is the depth weighted with stress, taking the same value common to all slices.

\[
\log \frac{1}{S} = \frac{\sum \frac{(\tau - \tau_u)}{z^*} \cdot \Delta V}{V_r} \cdot N^\epsilon \cdots \cdots \cdots \ (2)
\]

\[
\sum \frac{z_i (\tau_i - \tau_u) \cdot \Delta V_i}{V_r} = \sum \frac{z_i (\tau_i - \tau_u) \cdot \Delta V_i}{V_r} \cdots \cdots \cdots \ (3)
\]

**Model-D**

This model is worked out to allow one to consider stress distribution on the basis of the LP theory \(^{13}\). (See Fig. 5.) The portion below the surface is not divided in the direction of depth but in the direction of axis only, with disk-shaped slices formed after division. The life of a slice is calculated using the maximum amplitude of stress to which a slice is subjected \( \tau_0 \) and its depth \( z_0 \), and combining the life of each slice. In this way, the significance of the progress of cracking in the LP theory as it is can be brought into each slice. \( a_i \) denotes the width of each slice.

\[
\log \frac{1}{S} = \frac{1}{S} \sum \frac{\tau_0}{z_0} \cdot \Delta V_r \cdot N^\epsilon \cdot \sum \frac{\tau_0}{z_0} \cdot \Delta V = \sum \frac{\tau_0}{z_0} \cdot a_i \cdot \ell \cdot N^\epsilon \cdots \cdots \cdots \ (4)
\]

Fig. 6 shows the results of the life calculation according to the four models described above. Fig. 6 shows the ratios of life with reference to V1. The scale graduations are provided up to 20; the scale is exceeded depending on the model.
In the results of life tests shown in Fig. 6, model A capture overall trends in samples V2 and V5, while the agreement deteriorates in samples V3, V4, V7, and V8. Model B shows a similar trend, with the agreement becoming poorer in samples V3, V4, V5, V7, and V8. By contrast, model C in which the fatigue limit is not specified shows a better agreement than model B. When the fatigue limit is taken into consideration as in model B, the value of the stress term of equation (2), \( \tau_i, - \tau_u \), decreases as a result of subtracting the fatigue limit component \( \tau_u \) from the actual stress; accordingly, it is considered that the agreement with the experimental values becomes poor. Model D shows a trend similar to that of model C for which the fatigue limit is not considered. Table 2 brings together the evaluations of the results of calculation based on the four models described above. Of the results of calculation using the four life models above, those obtained from model A, model C, and model D show a relatively satisfactory agreement with experimental values, but the agreement in all cases cannot be said to be enough. Models C and D are ones that combine the life values of divided slices; in a case in which the edge load is strict as in sample V5, however, the calculated life becomes larger than the life obtained through experiments. On the other hand, model A, which does not combine life values of divided slices, captures the results of experiments on sample V5 best. This suggests that the method model A follows is effective in making it easier to reflect the effect of the edge on the life. The reason for a poorer agreement with experiments in models C and D with severe edge load is considered in the following.

### Table 2 Validation of the 4 models

<table>
<thead>
<tr>
<th></th>
<th>V2</th>
<th>V3</th>
<th>V4</th>
<th>V5</th>
<th>V6</th>
<th>V7</th>
<th>V8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model-A</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>△</td>
<td>△</td>
<td>△</td>
<td>×</td>
</tr>
<tr>
<td>Model-B</td>
<td>○</td>
<td>×</td>
<td>×</td>
<td>△</td>
<td>△</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Model-C</td>
<td>△</td>
<td>△</td>
<td>△</td>
<td>×</td>
<td>○</td>
<td>○</td>
<td>○</td>
</tr>
<tr>
<td>Model-D</td>
<td>○</td>
<td>△</td>
<td>△</td>
<td>×</td>
<td>○</td>
<td>○</td>
<td>△</td>
</tr>
</tbody>
</table>

○ : Errors from experimental values up to about 30%
△ : Errors from experimental values from 30% to twice
× : Errors from experimental values above twice

4. Discussion

The distribution in the axial direction of the life of each slice in model D is investigated on sample V5. Life values represented by LIFE Parameter shown in equation (5) are shown in Fig. 7. As shown in the comparison with equation (4), this value is perfectly proportional to the life. In equation (5), the stress is expressed in MPa and the length in mm.

\[
LIFE \ Parameter = \frac{\tau}{\tau_0} \cdot \frac{h-1}{z_0} \cdot \frac{1}{d_i} 
\]

In Fig. 7, intervals between calculation points (slice widths) are shortened, as shown in Fig. 3, to express the abrupt change in stress at the edge. In Fig. 7, discontinuity appears in the section near the edge in which the slice width is small \((x = 0.49–0.65 \text{ mm})\) and stress is concentrated; however, the life is not shortened so much as in the middle section. The reason for this is as follows: the slice width is reduced to one tenth of that in the middle in order to express stress concentration; as a result the value in equation (5) is lowered, and taking the reciprocal causes the life to be lengthened. In this way, the effect of the volume on the life is very great, with a slice with a smaller volume having a longer life. The life of the edge load section must become short basically; however, it has already become equal to that of the middle section at the stage shown in Fig. 7. However, combining them to determine the life as a whole further weakens the effect of the edge load. In other words, with the volume in the middle far greater than that of the edge, the effect of a decrease in the life in the edge is thinned out in the process of combining life values.
To express the detail mentioned above numerically, combining LIFE Parameter for the range $x = 0.5$–$0.64$ mm (in the edge) yields $2.95 \times 10^{-29}$, while doing the same operation for the range $x = 0.8$–$1.12$ mm yields $1.08 \times 10^{-29}$ (in the middle). This means that the life of the middle section is reduced to one third of that in the edge section in terms of calculation.

As seen in Table 1, however, sample V5 shows that 13 of 15 actual breakages have occurred in the edge section; therefore, the life of the edge section should be shorter than that in the middle section. As shown above, the method shown above does not agree with the real phenomenon at all. This is considered to be an adverse effect of combining life values of slices. This mechanism is common to methods of using stress distribution to combine life values. It is considered that a similar mechanism works when the calculated life is longer than the experimental life of sample V5 in model C based on the IH theory.

By the way, when a notched flat sheet undergoes the fatigue test, rupture starts at the notch section on which stress concentrates; for this reason, only the stress exerted on that section is considered and not combined with the life value calculated from the stress exerted on other parts. Assuming that most of cracks occur at the location at which the maximum amplitude of shearing stress occurs, the LP theory makes calculation using the stress and its depth at that location only. In other words, the LP theory practices the very calculating method just mentioned above. It is considered that the stress volume $V_i$ in the LP theory is intended for the consideration of the effect of the length of the race, not for the consideration of stress distribution.

As described above, the method of model A that uses the value at the part in which stress is concentrated is effective in making it easier to reflect the effect of the edge on the life. However, model A needs to be improved because of the disagreement of the result of calculation with the result of experiments in other T.P.s such as samples V3, V4, and V8.

5. Improved model (Multi-point LP method)

On the basis of the consideration described above, an improved model was worked out. Following the concept of the LP theory, the improved model gives consideration only to the stress amplitude $\tau_{peak}$ and its depth $z_{peak}$ at several parts in which stress is concentrated, expressing the life of one of those parts in the following equation:

$$\ln \frac{1}{S_{peak}} = \frac{\tau_{0,peak}}{z_{0,peak}^{1-1}} \cdot \phi_i \cdot N_{peak} \cdot a^\ell \quad \cdots (6)$$

In the full-crowning in which the experiment was conducted, the parts in which stress is concentrated are two in the edge and one in the middle, with equation (6) applied to these three parts to calculate the life of different parts. Since the life is determined by the stress in the middle only when edge load is absent, this model is quite identical to the LP theory. The value of the life as a whole $L$ is determined by combining the life values of these parts in which stress is concentrated. The life value combining the life value of each of the three parts in which stress is concentrated under the full-crowning condition is given by equation (7). The three sum values for $i$ are associated with two locations in the edge and with one in the middle.

$$\left(1/ L \right)^i = \sum_{i=1}^{3} \left(1/ L_i \right)^i \quad \cdots (7)$$

$\phi_i$ in equation (6) is a coefficient to correct the life value depending on how uniform the stress is, being called the “stress uniformity coefficient.” The three parts in which stress is concentrated are identified on the assumption that the peak of stress amplitude is distinct; depending on conditions, however, the degree of stress concentration in the middle becomes lower, making a peak not distinct. To clarify this condition, Fig. 8 shows the axial distribution of the maximum values of shearing stress observed on samples V1, V4, and V5. Fig. 8 is equivalent to a

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**Fig. 8** Distributions of max orthogonal shear stress amplitudes
diagram that plots the values obtained by searching the maximum values of shearing stress amplitude in the direction of depth on the contour diagram shown in Fig. 8. Fig. 8 shows how the maximum stress amplitude that determines the life of each slice in model D is distributed in the axial direction.

Fig. 8 shows that sample V4 exhibits a considerably lower stress concentration in the middle than sample V1 and the stress distribution has been made considerably uniform. It is estimated that such state appears when the crowning profile comes close to the optimum shape. This corresponds to a case in which the radius of the notch is very large in the fatigue test on the notched flat sheet described earlier. The maximum stress values are about the same at any position in the axial direction, and it becomes impossible to predict where a crack starts depending on the unevenness in strength of the material. It is estimated that such state is always equal to unity. What matters is extending 1 mm from the roller end face was excluded. Although the roller chamfer of T.P. is 0.5 mm, the part avoiding being affected by the stress peak in the edge.

Stress uniformity coefficient $\phi_v$

As seen from equation (6), a $\phi_v$ value greater unity shortens the life. In the following, the approach to determine the value of this coefficient is described. In a stress distribution such as that observed in a normal Hertz contact, it is considered that a sufficient stress concentration occurs and a stress uniformity coefficient needs not considering; therefore, it is assumed that $\phi_v = 1$ holds. Since the degree of stress concentration on the edge is more intense than in the middle, $\phi_v$ is always equal to unity. What matters is the stress in the middle, and as this becomes increasingly uniform, the value of $\phi_v$ increases.

**Fig. 9** shows diagrammatically the axial distributions of stress amplitude shown in **Fig. 8** into four types. Sample V1 provides a Hertz contact without edge load, corresponding to a quadratic stress distribution shown in **Fig. 9 (a)**. As described above, the $\phi_v$ value in this case is unity. As edge load is intensified, the stress distribution changes to those shown in **Figs. 9 (b) and (c)** and finally to that shown in **(d)**; in **Fig. 9 (d)**, stress concentration is not regarded as existing in the middle, and for this reason, the stress in the middle is neglected. However, since the edge exhibits a remarkable decrease in the life than the middle, combining life values including that in the middle yields about the same result. Sample V4 shows a characteristic relatively close to that shown in **Fig. 9 (c)**. Sample V5 shows a characteristic similar to that shown in **Fig. 9 (d)**. It may be better to express the degree of stress concentration by means of the curvature in the middle of the stress distribution shown in **Fig. 8**. However, this needing complex calculation, the degree of stress concentration is expressed by the area occupied by the stress in **Fig. 8**. This area is given by equation (8).

$$S = \sum \tau_{sj} \Delta a_j \quad \text{(8)}$$

The extent for which area $S$ is calculated is determined with the edge domain excluded in order to avoid being affected by the stress peak in the edge. Although the roller chamfer of T.P. is 0.5 mm, the part extending 1 mm from the roller end face was excluded in the present discussion.

In general, area $S$ is between area $S_a$ shown in **Fig. 9 (a)** and area $S_c$ shown in **Fig. 9 (c)**, with areas $S_a$ and $S_c$ being expressed by equations (9) and (10), respectively. $\phi_v$ is set at unity in **Fig. 9 (a)** and at 2 in **Fig. 9 (c)**, and $\phi_v$ values between unity and 2 are determined depending on area $S$. Therefore, $\phi_v$ can be expressed as shown in equation (11):

**Fig. 9 Type of max. shear stress distribution at center part**
6. Results of calculation using the improved model

Fig. 10 shows the result of calculation using the improved model. The illustration shows the result of calculation not using the stress uniformity coefficient \( \phi_v = 1 \) and that of calculation using model D. Table 3 shows the stress uniformity factors used in calculation.

It is clearly seen that the improved method exhibits a far better agreement with experimental results than the results obtained on four models (Fig. 6). In particular, the improved model excels other four models in that it can be used to determine the optimum crowning radius. In other words, the optimum crowning radius under a load of 13.72 kN can be determined using the results of the life test on samples V1 to V5. Ito and Sugiura consider that the optimum crowning radius is 830 mm, slightly closer to the result from sample V2 than to that from sample V3, while the result from sample V4 shows a peak with models A, B, C, and D. On the other hand, the peak life value appears between samples V2 and V3, capturing the life value peak in the experimental results successfully. Table 4 shows the result of evaluation of the improved method in comparison with the result obtained using model D.

A shortcoming with the improved method is the estimation of the calculated life value at a lower value compared with the experimental value under severe edge load condition as in sample V5. This is due to the fact that despite a change in the stress value due to the plastic deformation caused by the high contact pressure exerted by edge load, the calculation of stress values on the basis of the theory of elasticity has yielded an estimation of a higher stress value than an actual one. For this reason, the estimation is on the safe side from the design viewpoint, and this will pose no practical problem.

7. Afterword

Methods of predicting the life under the effect of edge load occurring in the roller bearing were discussed.

It was clarified that the commonly-used life calculation method of dividing subsurface volume into slices and calculating and combining the life values of individual divided slices does not yield a correct life prediction due to a great effect exerted by the stress volume.

As an improved method, the “multi-point LP method” was proposed in which, on the basis of an idea that each of several parts where stress is concentrated forms the starting point leading to bearing damage, the life value at each starting point to damage is calculated by the LP theory and the values thus calculated are combined. This method introduces the “stress uniformity factor” to take into consideration the phenomenon of fatigue-based life shortening similar to the dimension effect occurring when the stress distribution becomes uniform. To determine the stress uniformity factor, the method of determining it using the area formed by the axial distribution of the maximum values of shearing stress was employed temporarily. To determine this value, it will be necessary to accumulate experimental data as in the process in which the dimension factor was determined in the field of strength of materials in the past.

This paper has been prepared with additions and modifications on the basis of a paper titled “Improved Method of Roller Bearing Fatigue Life Prediction Under Edge Loading Conditions” (Tribology Transactions, Vol. 53, issue 5, pp. 695–702).
References


